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**LETTER TO THE EDITOR**

# Angular velocity in rotating systems

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**Abstract.** This letter indicates errors in the recent paper 'Relativity of rotation' by Browne, published in *Journal of Physics A: Mathematical and General.*

In a recent paper Browne (1977) discussed the application of relativity to rotating systems. He derived the transformations relating the angular velocity of the rotating system as measured within the rotating frame to the angular velocity measured in the laboratory frame. Assuming that the laboratory frame is stationary with respect to the universe (i.e. stationary with respect to the fixed stars) the equations he derived were

$$\omega_0 = \frac{\omega}{(1-\omega^2 r^2/c^2)^{1/2}} \qquad \omega = \frac{\omega_0}{(1+\omega_0^2 r^2/c^2)^{1/2}}$$

where $\omega$ is the angular velocity of the rotating system with respect to the laboratory frame, $\omega_0$ is the angular velocity of the laboratory frame with respect to the rotating frame, and $r$ is the radial distance to the rotating observer as measured in the laboratory frame.

These equations have previously been derived by Jennison (1964) and have subsequently been discussed by several authors (Davies and Jennison 1975, Ashworth and Jennison 1976, Davies 1976).

The rate of ticking of the local clock of a rotating observer is a function of radius, as has been shown by the Mossbauer experiments on rotating systems (Hay *et al* 1960, Champeney and Moon 1961, Kündig 1963, Champeney *et al* 1965, Farley *et al* 1968). It is evident that if $\omega_0$ is measured by the timing of successive transits of a line of meridian fixed in the laboratory frame, over which the rotating observer passes, then $\omega_0$ must also be a function of $r$. In contrast, the parameter $\omega$ will be constant if the rotating system is a synchronous one such as a rotating disc.

Browne makes the error of assuming $\omega_0$ to be constant and so, not surprisingly, with $\omega$ as a function of $r$, obtains different results from those of Jennison *et al* who assume $\omega$ to be constant. But if we do assume $\omega$ to be constant then the meaning of the Thomas precession as just the manifestation of the difference between the two angular velocities becomes easier to understand (Davies and Jennison 1975, Jennison 1977) i.e. $\omega_T = \omega - \omega_0$. It is, incidentally, not necessary to apply two Lorentz transformations to generate the Thomas precession as is normally done in standard texts (e.g. Eisberg 1967). The Thomas precession may be derived using only one transformation (Ashworth 1977).

Unfortunately, Browne's incorrect assumption that $\omega_0$ is constant has implications in the application of the Thomas precession and he concludes that this precession produces a 'differential rotation'. An experiment to measure this 'differential rotation' was proposed by Weinstein (1971) and the problem was subsequently discussed by Whitmire (1972a, b). The experiment was performed by Phipps (1974), with a negative result, showing that differential rotation did not occur.

Another result of Browne's incorrect assumption that $\omega_0$ is constant is that the paths of rays of light measured by rotating observers become Archimedian spirals instead of arcs of circles as derived by Jennison (1963). The description of the path of a ray of light across a rotating disc depends greatly upon the way in which measurements were made and interpreted. If measurements are made *in situ* at each point along the light path then the circular solution holds, but if this path is measured entirely by a single observer at the centre of rotation of the disc then it transforms to an Archimedian spiral. This problem has been discussed in detail by Ashworth and Davies (1977).

The argument about whether to define the angular velocity as constant in the laboratory frame or in the rotating frame may at first sight appear to be superfluous since general relativity tells us that we may analyse any problem in any coordinate system. However, only $\omega$ = constant can apply to those systems which are commonly considered as solid discs.

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